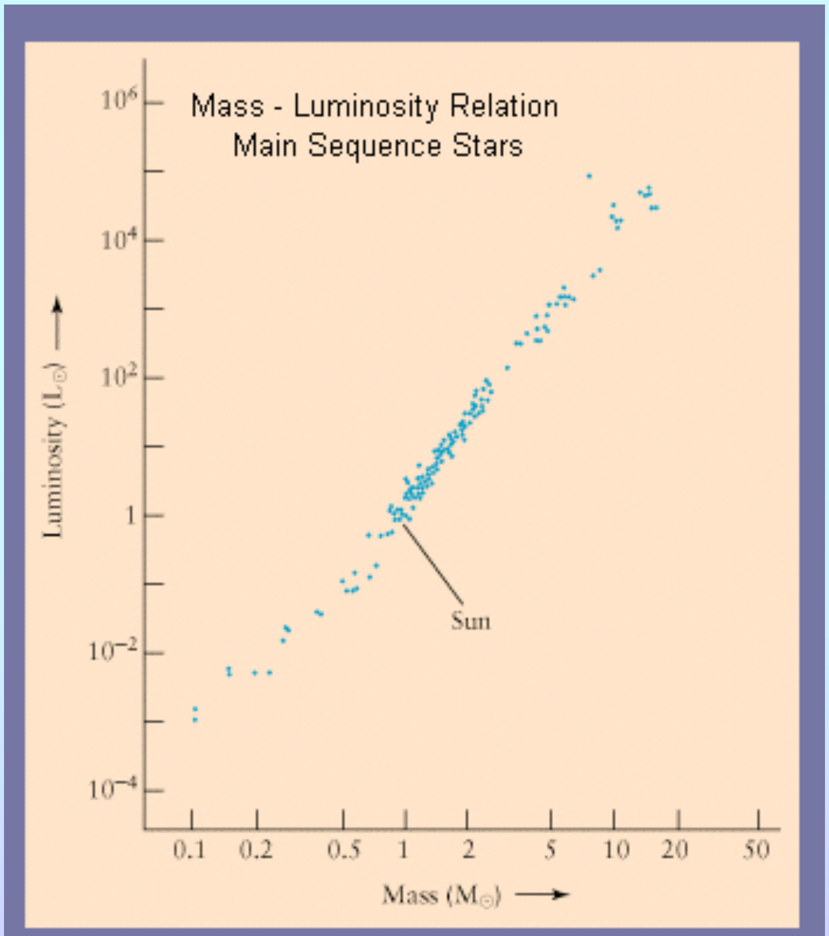
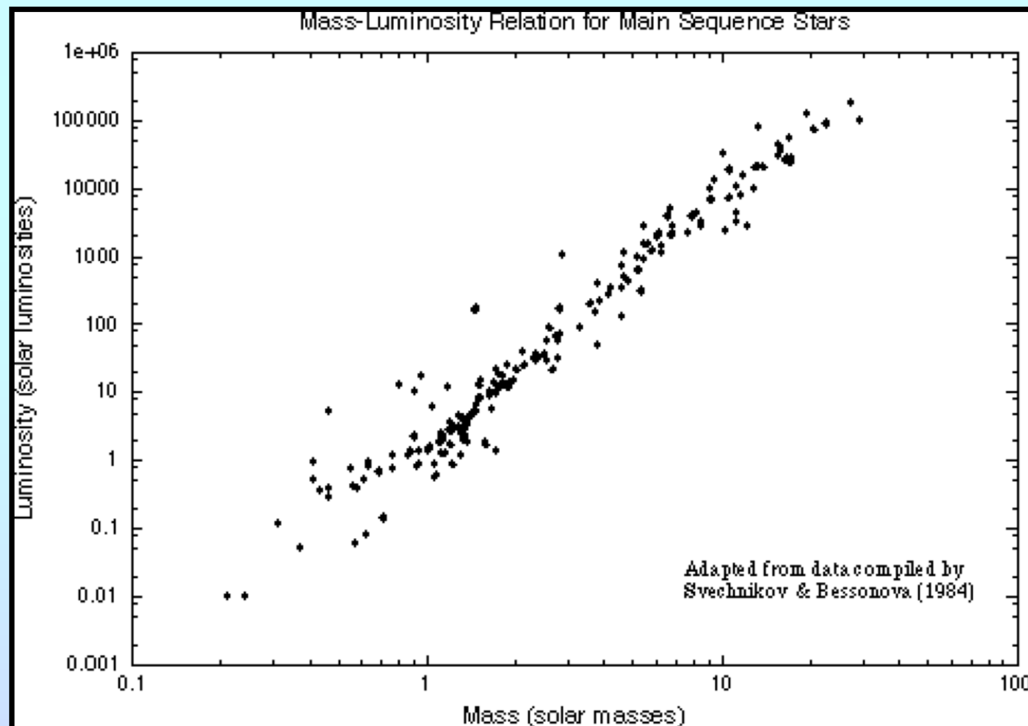
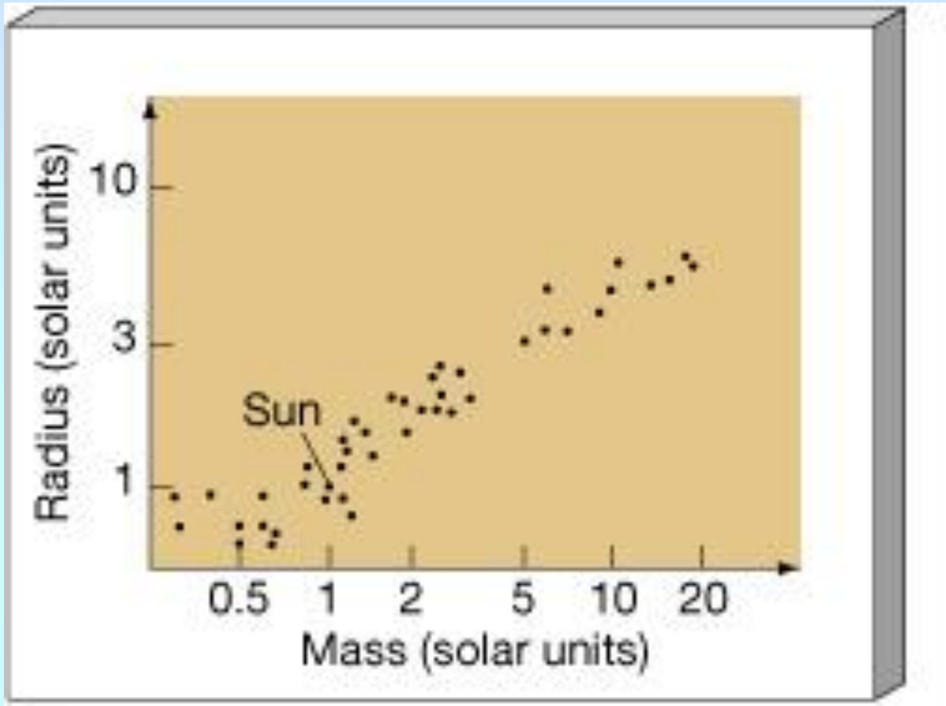
Stellar Masses and the Main Sequence

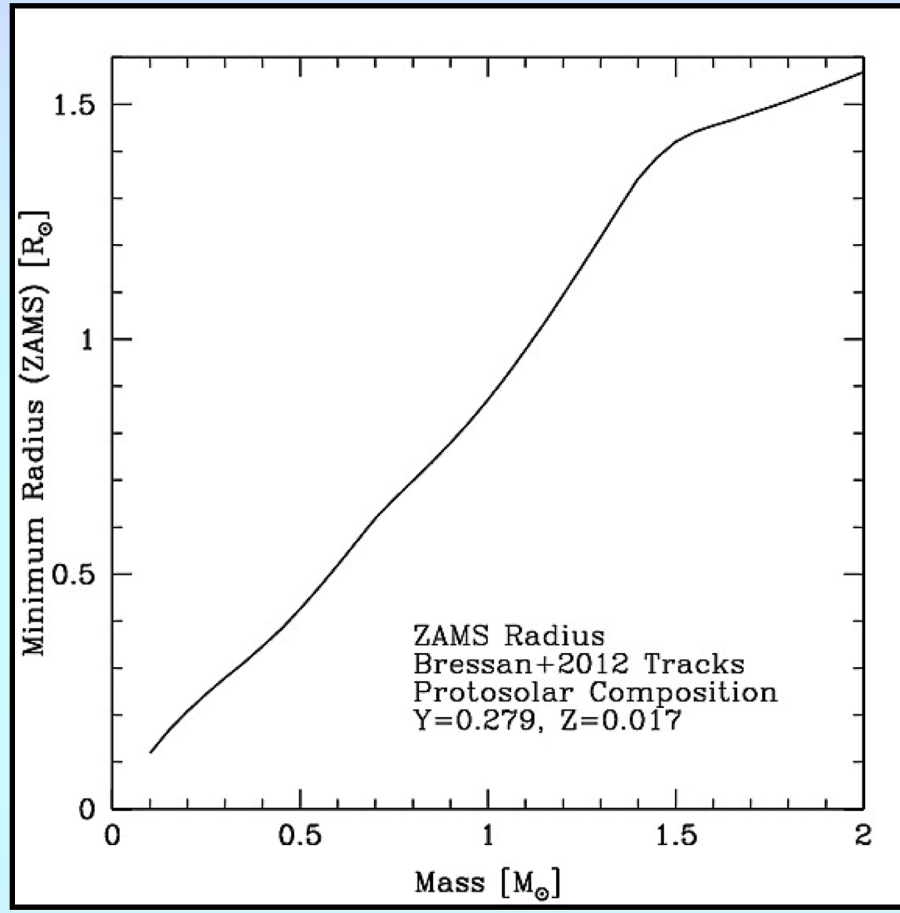
Measurements of main-sequence stars demonstrate that there is a mass-luminosity relationship, i.e., $L \propto M^{\eta}$. For $M > 1\ M_{\odot}$ $\eta \sim 3.88$, while at lower masses, the relation flattens out. A good rule-of-thumb is $L \propto M^{\eta}$, with $\eta \sim 3.5$.





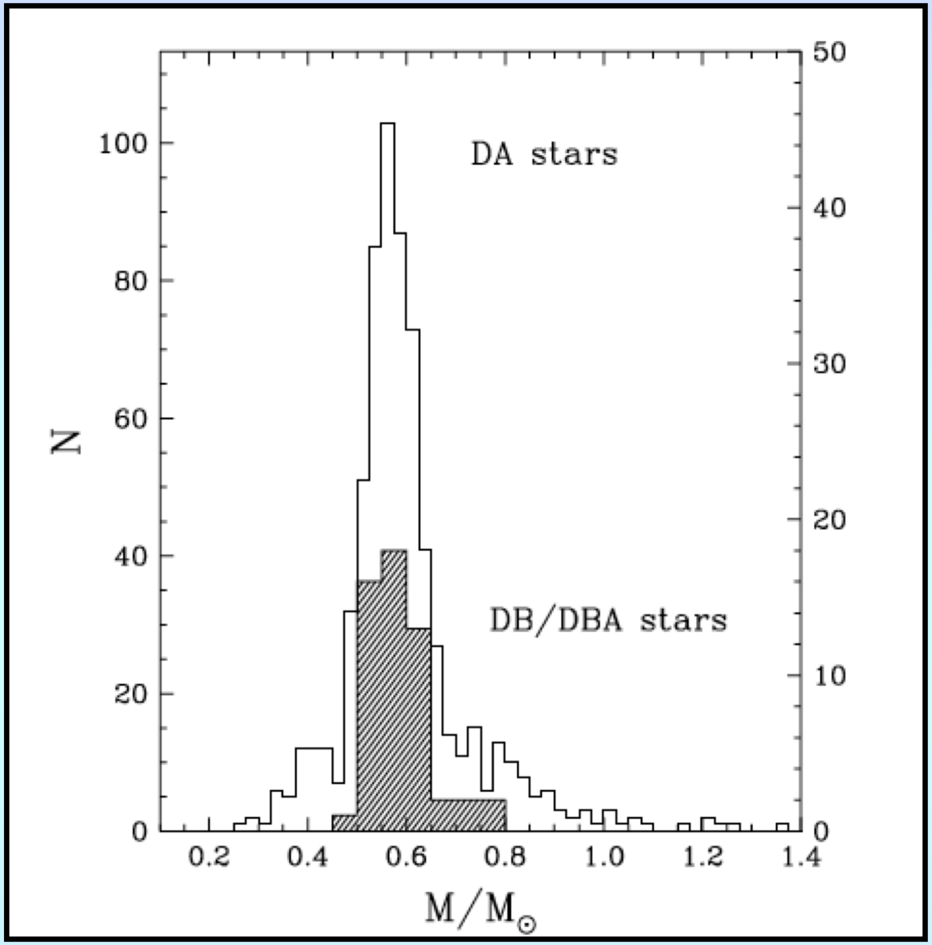
Main Sequence Mass-Radius Relation

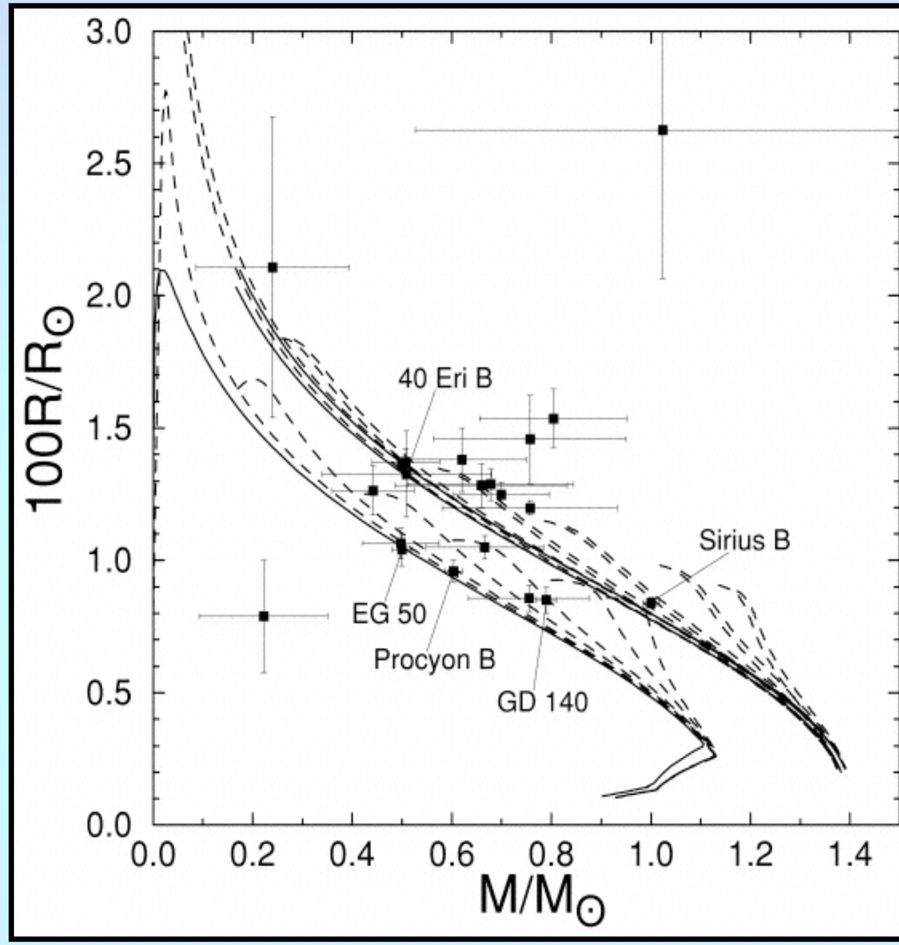




There is also a mass-radius relation for main-sequence stars. When parameterized via a power law, $R \propto M^{\xi}$, $\xi \sim 0.57$ for masses $M > 1\ M_{\odot}$, and $\xi \sim 0.8$ for $M < 1\ M_{\odot}$.

Stellar Masses for White Dwarfs





The masses of white dwarf stars are all less than 1.4 $M_\odot$. Most are ~ 0.59 $M_\odot$.

There is also an inverse mass-radius relation for white dwarfs. The simple theory says $M \propto R^\alpha$, with $\alpha = -1/3$.

Numbers to Keep in Mind

- $\tau_\odot$ ~ 10 Gyr = Main sequence lifetime of the Sun
- $1\ M_\odot \sim 2 \times 10^{33}$ gm = Mass of the Sun
- $1\ L_\odot \sim 4 \times 10^{33}$ ergs/sec = Luminosity of the Sun
- $1\ R_\odot \sim 7 \times 10^{10}$ cm = Radius of the Sun
- $Q \sim 6.3 \times 10^{18}$ ergs/gm = Energy from hydrogen fusion
- Δm ~ 27 MeV = mass defect for hydrogen fusion
- Δm ~ 0.7% = percent mass defect for hydrogen fusion
- X ~ 0.75 = fraction of hydrogen (by mass) in the Sun
- Y ~ 0.23 = fraction of helium (by mass) in the Sun
- Z ~ 0.02 = fraction of “metals” (by mass) in the Sun

Stellar Timescales

Before starting to discuss how stars work, it is important to have an order-of-magnitude feel for stellar timescales. They are the shortcut to everything!

- **The Nuclear Timescale**: How long will a star live? (In other words, how long will it take to use up its nuclear fuel?)

$$\tau_{\text{nuc}} = \frac{\text{Fuel}}{\text{Rate of Consumption}} = \frac{Q\,M}{L}$$

For hydrogen fusion, $Q = 6.3 \times 10^{18}$ ergs/gm. (If the star is fusing helium, the coefficient is an order of magnitude smaller.) Notes:

- Main sequence stars will only consume ~ 0.1 of their available fuel before they must adjust their structure.
- You can scale to the Sun: $\tau_{\text{nuc}} = 10^{10}$ years.

Stellar Timescales

- **The Thermal Timescale**: How long does it take for a star to adjust its structure? (How long does it take to release its energy?) This is also called the Kelvin-Helmholtz timescale

$$\tau_{\mathrm{KH}} = \frac{\text{Gravitational Energy}}{\text{Rate of Consumption}} = \frac{GM^2}{RL}$$

This describes how long a star can shine with no nuclear energy generation. Alternatively, it describes how long it takes energy produced by gravitational contraction to work its way out. For the Sun, this number is ~ 10^8 years.

Stellar Timescales

- **The Dynamical Timescale**: How long does it take a star's interior to feel changes at its surface? (For instance, if a star accretes mass, how long would it take the interior to feel the extra weight?) Alternatively, how long does it take an object to "fall" to the star's core (under constant g)?

$$\tau_{\text{dyn}} = \frac{\text{Size of the Star}}{\text{Speed of Pressure Wave}} = \left(\frac{R^3}{G\,M}\right)^{1/2} = \left(\frac{R}{g}\right)^{1/2}$$

Note: this is equivalent to the free-fall time, and it's also equivalent to Kepler's 3rd law. For the Sun, $\tau_{\text{dyn}} \sim 30$ minutes.

Stellar Timescales

- **The Mass-Loss Timescale**: How long does it take for a star to lose all its mass via its wind?

$$\tau_{ML} = \frac{\text{Mass of Star}}{\text{Mass Loss Rate}} = \frac{M}{\dot{M}}$$

For the Sun today, this is $\sim 10^{13}$ years.

Note: for most (but not all) stars, the nuclear, thermal, dynamical, and mass-loss timescales are very different. This makes stellar structure and evolution easy to model. The complications arise when two are more timescales become comparable.

Stellar Structure

The internal structure of most stars can be computed easily(?) by solving 4 simultaneous differential equations with 4 unknowns.

Mass Conservation: $$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

Momentum Conservation: $$\frac{dP(r)}{dr} = -g\rho = -\frac{GM(r)}{r^2}\rho(r)$$

Energy Conservation: $$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\varepsilon(\rho,T)$$

Thermal Structure: $$\frac{dT(r)}{dr} = \frac{dP(r)}{dr}\frac{dT}{dP} = \frac{dP}{dr}\frac{T}{P}\left(\frac{d\ln T}{d\ln P}\right) = \frac{dP}{dr}\frac{T}{P}\nabla$$

where $P = P(\rho,\mu,T)$

$$\varepsilon(\rho,T) = \varepsilon_{\text{nuclear}} - \varepsilon_{\text{neutrino}} + \varepsilon_{\text{gravitational}} = \text{energy generation}$$

$$\nabla = \frac{d\ln T}{d\ln P} = \frac{3\kappa(\rho,T)LP}{16\pi acGMT^4} \text{ or } \nabla_{\text{ad}}$$

$$\kappa(\rho,T) = \text{opacity}$$

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Disclaimer: I wrote these equations for easy of understanding and used radius as the independent variable. Real calculations of stellar structure use mass as the independent variable.

Digression: Mean Molecular Weight

Mean molecular weight (μ) is defined as mass per unit mole of material, or, alternatively, the mean mass of a particle in Atomic Mass Units. Thus, number density is related to the mass density, ρ, by

$$n = \frac{\rho}{\mu m_a} = \frac{\rho N_{\mathrm{avo}}}{\mu}$$

Often, number density is split into two terms, one for ions

$$n_I = \sum_i n_i = \rho N_{\mathrm{avo}} \sum_i \frac{x_i}{A_i} = \frac{\rho N_{\mathrm{avo}}}{\mu_{\mathrm{I}}} \quad \text{where} \quad \mu_I = \left(\sum_i \frac{x_i}{A_i} \right)^{-1}$$

And one for the (massless) electrons

$$n_e = \rho N_{\mathrm{avo}} \sum_i \frac{x_i}{A_i} f_i Z_i = \frac{\rho N_{\mathrm{avo}}}{\mu_{\mathrm{e}}} \quad \text{where} \quad \mu_e = \left(\sum_i \frac{Z_i x_i f_i}{A_i} \right)^{-1}$$

where x_i is the mass fraction of the species, Z_i is its atomic number, A_i, its atomic mass, and f_i, the fraction of electrons that are free.

Mean Molecular Weight -- Simplification

In stars, the expressions for mean molecular weight can be greatly simplified. For example, the fraction of heavy elements in a star is small ($Z \sim 0.02$), so

$$\mu_I = \left(\sum_i \frac{x_i}{A_i} \right)^{-1} = \left(\frac{X}{1} + \frac{Y}{4} + \frac{Z}{\sim 14} \right)^{-1} \approx \left(X + \frac{1-X}{4} \right)^{-1} = \frac{4}{1+3X}$$

Also, if we assume that the gas in the interior of a star is (almost) entirely ionized, $f_i \sim 1$. And, while hydrogen has $Z_i/A_i = 1$, most other elements have $Z_i/A_i \sim ½$. So

$$\mu_e = \left(\sum_i \frac{Z_i x_i f_i}{A_i} \right)^{-1} = \left(X + \frac{1}{2}Y + \frac{1}{2}Z \right)^{-1} = \left(X + \frac{1-X}{2} \right)^{-1} = \frac{2}{1+X}$$

The combined mean molecular weight is therefore

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_e}$$

Equation of State

To solve the equations of stellar structure, one must also known the relationship between pressure, density, and temperature and mean molecular weight. This is called the equation of state. The pressure comes from 3 sources, i.e., $P = P_{\text{rad}} + P_{\text{ion}} + P_{\text{electron}}$.

- Radiation pressure: $P_{\text{rad}} = \frac{1}{3} a T^4$

- Ion pressure (ideal gas): $P_{\text{ion}} = \frac{\rho}{\mu_i m_a} kT$

- Electron pressure: In most main sequence stars, electrons act as an ideal gas, but in the cores of giants and very low-mass main-sequence stars, partial degeneracy can occur. One must then solve Fermi-Dirac integrals, i.e.,

$$P_e = \frac{4\pi}{3h^3} \frac{(2m_e kT)^{5/2}}{m_e} \int_0^\infty \frac{\eta^{3/2}}{1 + e^{\eta - \psi}} d\eta \quad \text{where} \quad \psi = \ln\left\{ \frac{n_e h^3}{2(2\pi m_e kT)^{3/2}} \right\}$$

Opacity

Opacity is the opposite of transmittance. To get the appropriate (Rosseland) mean, you have to integrate over the blackbody function. Opacity is given as digital tables in temperature and density, but there are rough approximations:

- Electron (Thomson) scattering:

$$\kappa_e = \frac{n_e \sigma_e}{\rho} = 0.2(1+X)\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

- Free-free absorption (Kramer's style opacity):

$$\kappa_{\mathrm{ff}} \sim 10^{23} \frac{Z^2}{\mu_e \mu_I} \rho T^{-7/2}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

- Bound-free absorption (Kramer's style opacity):

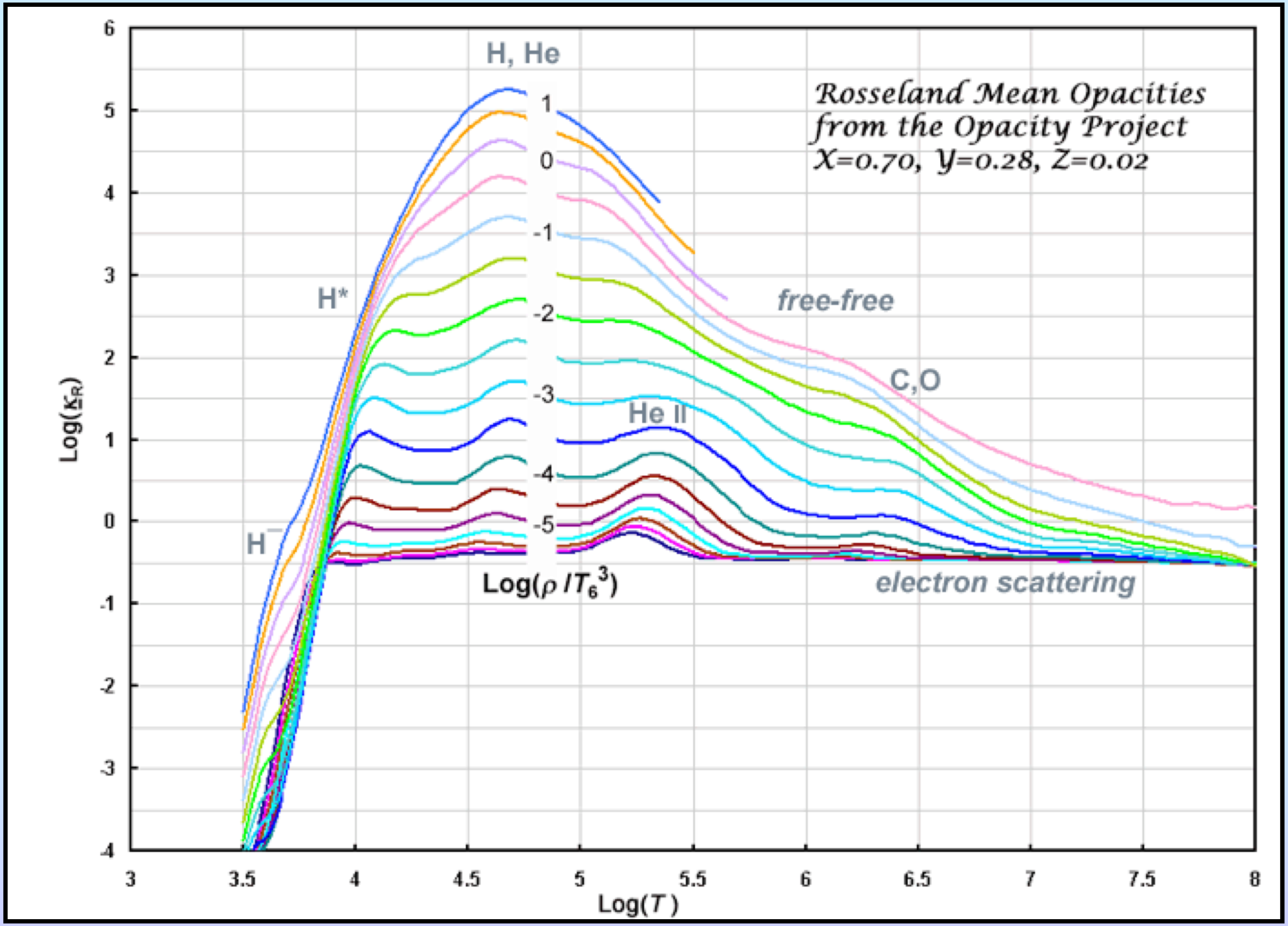
$$\kappa_{\mathrm{bf}} \sim 10^{25} (1+X) \rho T^{-7/2}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

- Bound-bound absorption (complex, but roughly Kramers)
- H^- opacity ($H^- + h\nu \leftrightarrows H + e^-$): complicated, but below $\sim 10^4$ K

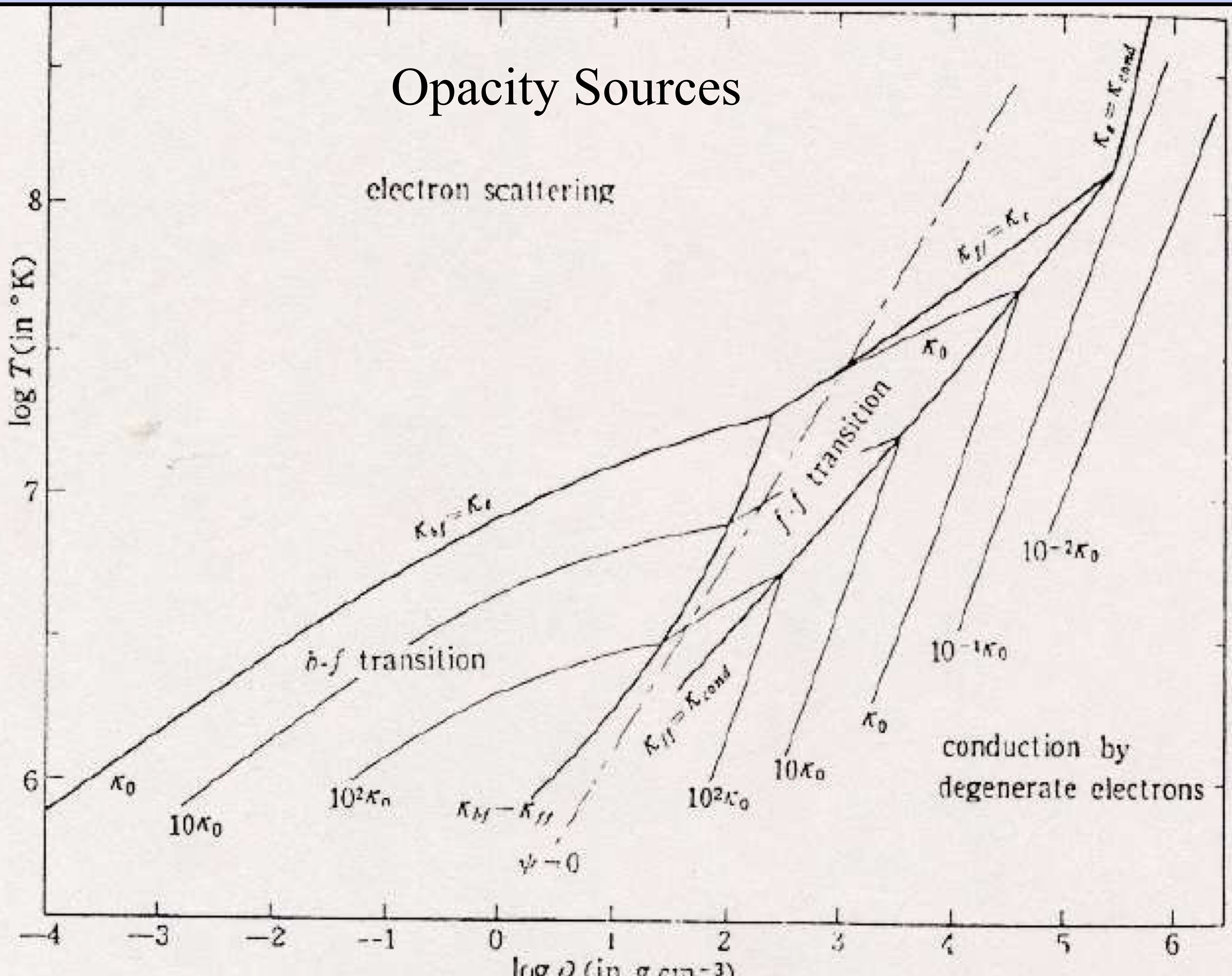
$$\kappa_{H^-} \sim 2.5 \times 10^{-31} (Z/0.02)\ \rho^{1/2}\ T^9\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

Opacity

Massive supercomputer projects have calculated Rosseland mean opacities as a function of density and temperature.



Opacity Sources



Energy Transport: Radiation

In theory, there are 3 ways to transport heat:

- Conduction: Only important in white dwarfs and neutron stars
- Convection: mixing
- Radiation: diffusion of light due to absorption and re-emission.

Radiation is nothing more than the random walk of energy, where the mean free path is $l_{\mathrm{ph}} = 1/\kappa\rho$. The expression for energy transport of this type is fairly simple:

$$F = \frac{L}{4\pi r^2} = -\frac{1}{3} c\, l_{\mathrm{ph}} \frac{dU}{dr} = -\frac{1}{3} c\, l_{\mathrm{ph}} \left(4\pi T^3 \frac{dT}{dr} \right)$$

This is usually re-written using thermodynamical relations to

$$\frac{dT}{dr} = \frac{GM\rho T}{R^2 P} \nabla_{\mathrm{rad}} \quad \text{where} \quad \nabla_{\mathrm{rad}} = \frac{3\kappa L P}{16\pi a c G M T^4}$$

Energy Transport: Convection

In general, convection is more efficient, *if* it exists. If a blob of material is displaced outward, does it continue to rise (taking its heat with it) or sink back to where it started? In other words, at location $r + \delta r$, is the blob denser or less dense than its new environment?

Therefore, for convection to occur $\left(\frac{d\rho}{dr}\right)_{\text{blob}} < \left(\frac{d\rho}{dr}\right)_{\text{star}}$

After a bit of thermodynamic manipulation, this becomes

$$\frac{\partial \ln T}{\partial \ln P} = \nabla_{\text{ad}} < \nabla_{\text{rad}}$$

Note: ∇_{ad} is a thermodynamic quantity intrinsic to the gas. For an ideal gas ($P \propto \rho^{\gamma}$, with $\gamma=5/3$), $\nabla_{\text{ad}} = 0.4$. For pure radiation pressure ($P \propto \frac{1}{3} aT^4$), $\nabla_{\text{ad}} = 0.25$.

Nuclear Reaction Rates

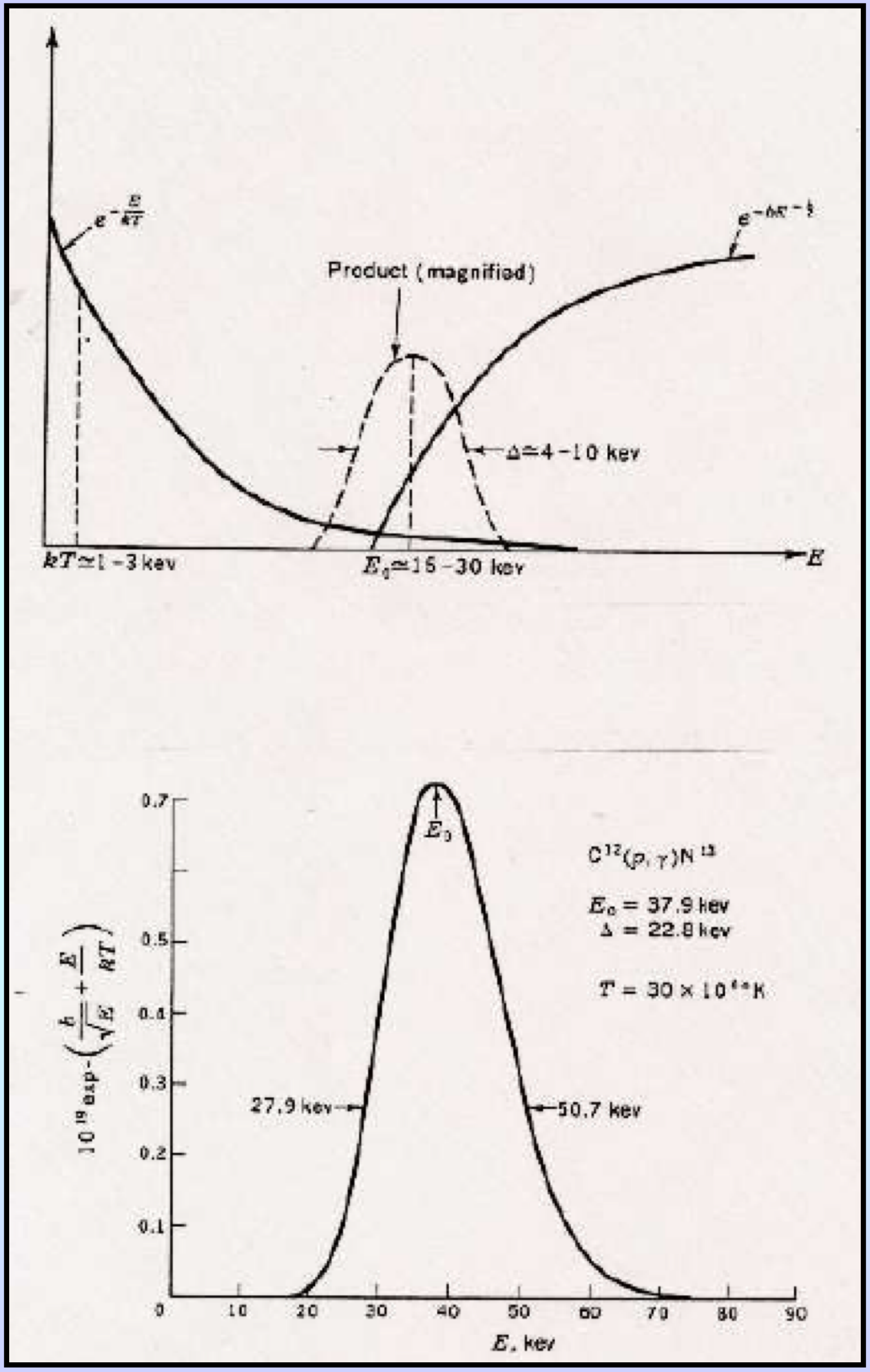
Most light-element nuclear reactions depend on density (squared) and temperature to some power. To compute that power, one has to integrate reaction rate cross-sections over the Maxwellian distribution.

The cross sections are energy dependent, since they depend on momentum and tunneling.

$$\sigma \propto \pi \lambda^2 \propto \left(\frac{1}{p^2}\right) \propto \frac{1}{E}$$

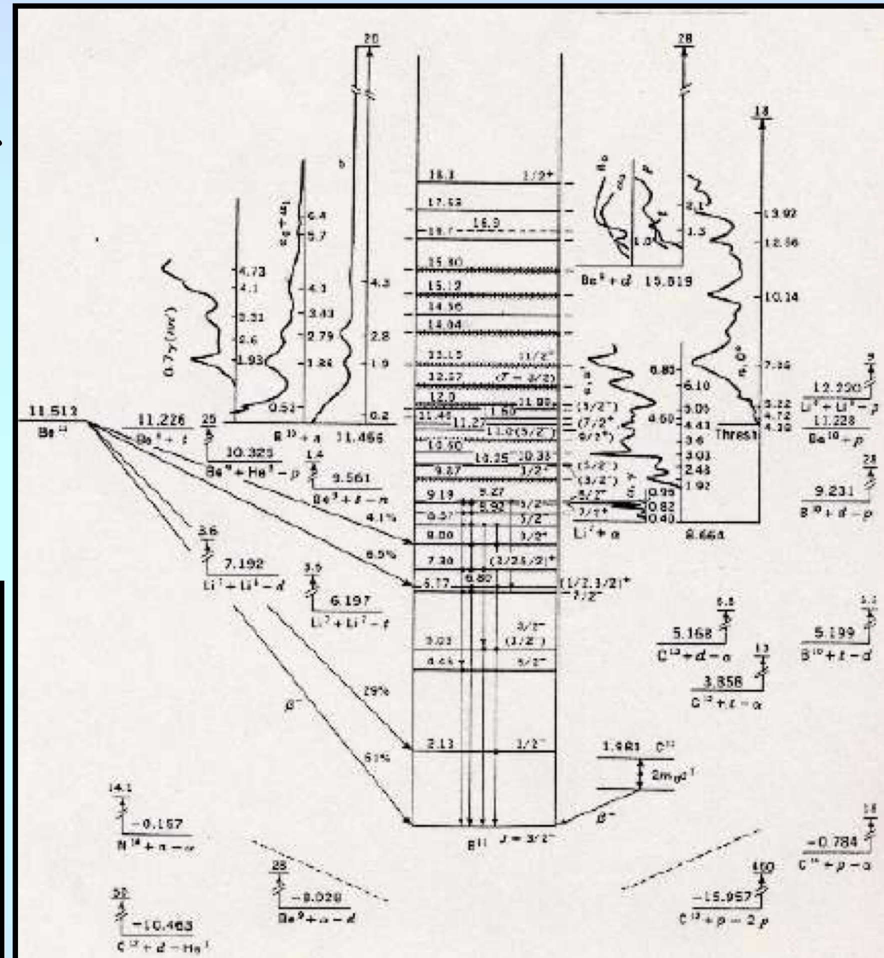
$$\sigma \propto \exp\left\{-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right\} \propto \exp\left\{-\frac{KZ_1 Z_2}{E^{1/2}}\right\}$$

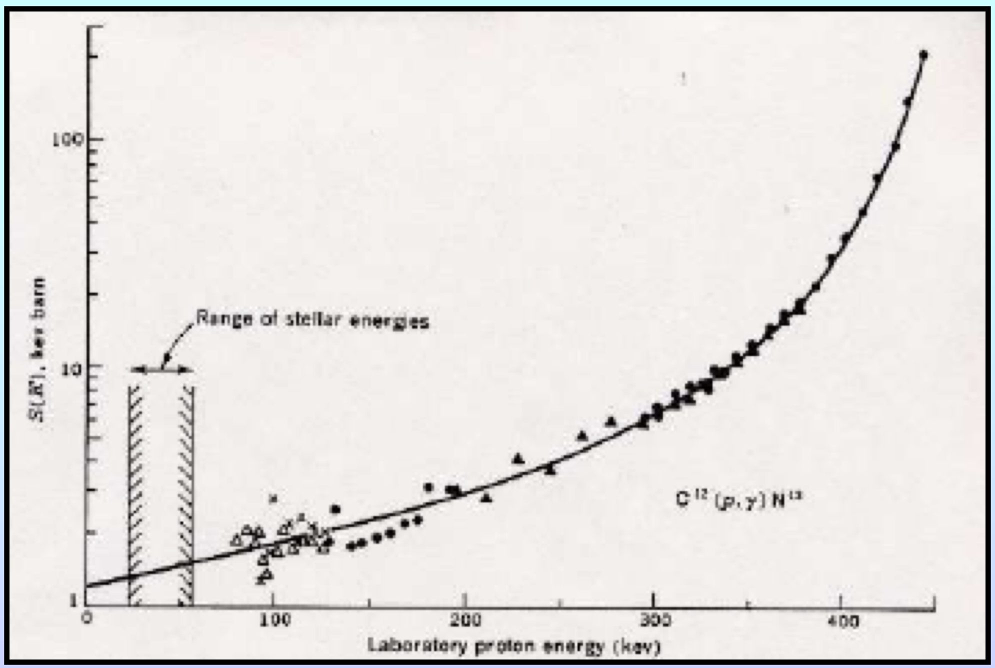
$$\langle \sigma v \rangle = \frac{14.16}{T_6^{1/3}} \left(Z_1^2 Z_2^2 A\right)^{1/3} - \frac{2}{3}$$



Nuclear Reaction Rates

Most heavy-element nuclear reactions involve nuclear resonances, many of which are poorly known. This introduces uncertainty into some aspects of heavy element nucleosynthesis.





Proton-Proton Chain

The Sun produces energy by fusing 4 hydrogen atoms into one helium atom, thereby creating 26.73 MeV per helium nucleus. Most of this happens via the proton-proton chain.

- PP I:
  - $^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu_e$ $\langle\varepsilon_\nu\rangle = 0.263$ MeV
  - $^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma$
  - $^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H$
- PP II:
  - $^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$
  - $^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e + \gamma$ $\langle\varepsilon_\nu\rangle = 0.8$ MeV
  - $^{7}Li + {}^{1}H \rightarrow {}^{4}He + {}^{4}He$
- PP III:
  - $^{7}Be + {}^{1}H \rightarrow {}^{8}B + \gamma$
  - $^{8}B \rightarrow {}^{8}Be + e^{+} + \nu_e$ $\langle\varepsilon_\nu\rangle = 7.2$ MeV
  - $^{8}Be \rightarrow {}^{4}He + {}^{4}He$

PERIODIC TABLE
Atomic Properties of the Elements

NIST
National Institute of Standards and Technology
U.S. Department of Commerce

Physical Measurement Laboratory
www.nist.gov/pml

Standard Reference Data
www.nist.gov/srd

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.626 07 x 10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.602 177 x 10^{-19} C	
electron mass	m_e	9.109 38 x 10^{-31} kg	
	$m_e c^2$	0.510 999 MeV	
proton mass	m_p	1.672 622 x 10^{-27} kg	
fine-structure constant	α	1/137.035 999	
Rydberg constant	R_∞	10 973 731.569 m^{-1}	
	$R_\infty c$	3.289 841 960 x 10^{15} Hz	
	$R_\infty hc$	13.605 69 eV	
Boltzmann constant	k	1.380 6 x 10^{-23} J K^{-1}	

- Solids
- Liquids
- Gases
- Artificially Prepared

Period	Group 1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIII	9 VIII	10 VIII	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 $^2S_{1/2}$ **H** Hydrogen 1.008* $1s$ 13.5984																	2 $^1S_0$ **He** Helium 4.002602 $1s^2$ 24.5874
2	3 $^2S_{1/2}$ **Li** Lithium 6.94* $1s^2 2s$ 5.3917	4 $^1S_0$ **Be** Beryllium 9.0121831 $1s^2 2s^2$ 9.3227											5 $^2P^\circ_{1/2}$ **B** Boron 10.81* $1s^2 2s^2 2p$ 8.2980	6 $^3P_0$ **C** Carbon 12.011* $1s^2 2s^2 2p^2$ 11.2603	7 $^4S^\circ_{3/2}$ **N** Nitrogen 14.007* $1s^2 2s^2 2p^3$ 14.5341	8 $^3P_2$ **O** Oxygen 15.999* $1s^2 2s^2 2p^4$ 13.6181	9 $^2P^\circ_{3/2}$ **F** Fluorine 18.99840316 $1s^2 2s^2 2p^5$ 17.4228	10 $^1S_0$ **Ne** Neon 20.1797 $1s^2 2s^2 2p^6$ 21.5645
3	11 $^2S_{1/2}$ **Na** Sodium 22.98976928 [Ne]$3s$ 5.1391	12 $^1S_0$ **Mg** Magnesium 24.305* [Ne]$3s^2$ 7.6462											13 $^2P^\circ_{1/2}$ **Al** Aluminum 26.9815385 [Ne]$3s^2 3p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.085* [Ne]$3s^2 3p^2$ 8.1517	15 $^4S^\circ_{3/2}$ **P** Phosphorus 30.97376200 [Ne]$3s^2 3p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.06* [Ne]$3s^2 3p^4$ 10.3600	17 $^2P^\circ_{3/2}$ **Cl** Chlorine 35.45* [Ne]$3s^2 3p^5$ 12.9676	18 $^1S_0$ **Ar** Argon 39.948 [Ne]$3s^2 3p^6$ 15.7596
4	19 $^2S_{1/2}$ **K** Potassium 39.0983 [Ar]$4s$ 4.3407	20 $^1S_0$ **Ca** Calcium 40.078 [Ar]$4s^2$ 6.1132	21 $^2D_{3/2}$ **Sc** Scandium 44.955908 [Ar]$3d4s^2$ 6.5615	22 $^3F_2$ **Ti** Titanium 47.867 [Ar]$3d^2 4s^2$ 6.8281	23 $^4F_{3/2}$ **V** Vanadium 50.9415 [Ar]$3d^3 4s^2$ 6.7462	24 $^7S_3$ **Cr** Chromium 51.9961 [Ar]$3d^5 4s$ 6.7665	25 $^6S_{5/2}$ **Mn** Manganese 54.938044 [Ar]$3d^5 4s^2$ 7.4340	26 $^5D_4$ **Fe** Iron 55.845 [Ar]$3d^6 4s^2$ 7.9025	27 $^4F_{9/2}$ **Co** Cobalt 58.933194 [Ar]$3d^7 4s^2$ 7.8810	28 $^3F_4$ **Ni** Nickel 58.6934 [Ar]$3d^8 4s^2$ 7.6399	29 $^2S_{1/2}$ **Cu** Copper 63.546 [Ar]$3d^{10} 4s$ 7.7264	30 $^1S_0$ **Zn** Zinc 65.38 [Ar]$3d^{10} 4s^2$ 9.3942	31 $^2P^\circ_{1/2}$ **Ga** Gallium 69.723 [Ar]$3d^{10} 4s^2 4p$ 5.9993	32 $^3P_0$ **Ge** Germanium 72.630 [Ar]$3d^{10} 4s^2 4p^2$ 7.8994	33 $^4S^\circ_{3/2}$ **As** Arsenic 74.921595 [Ar]$3d^{10} 4s^2 4p^3$ 9.7886	34 $^3P_2$ **Se** Selenium 78.971 [Ar]$3d^{10} 4s^2 4p^4$ 9.7524	35 $^2P^\circ_{3/2}$ **Br** Bromine 79.904* [Ar]$3d^{10} 4s^2 4p^5$ 11.8138	36 $^1S_0$ **Kr** Krypton 83.798 [Ar]$3d^{10} 4s^2 4p^6$ 13.9996
5	37 $^2S_{1/2}$ **Rb** Rubidium 85.4678 [Kr]$5s$ 4.1771	38 $^1S_0$ **Sr** Strontium 87.62 [Kr]$5s^2$ 5.6949	39 $^2D_{3/2}$ **Y** Yttrium 88.90584 [Kr]$4d5s^2$ 6.2173	40 $^3F_2$ **Zr** Zirconium 91.224 [Kr]$4d^2 5s^2$ 6.6339	41 $^6D_{1/2}$ **Nb** Niobium 92.90637 [Kr]$4d^4 5s$ 6.7589	42 $^7S_3$ **Mo** Molybdenum 95.95 [Kr]$4d^5 5s$ 7.0924	43 $^6S_{5/2}$ **Tc** Technetium (98) [Kr]$4d^5 5s^2$ 7.1194	44 $^5F_5$ **Ru** Ruthenium 101.07 [Kr]$4d^7 5s$ 7.3605	45 $^4F_{9/2}$ **Rh** Rhodium 102.90550 [Kr]$4d^8 5s$ 7.4589	46 $^1S_0$ **Pd** Palladium 106.42 [Kr]$4d^{10}$ 8.3369	47 $^2S_{1/2}$ **Ag** Silver 107.8682 [Kr]$4d^{10} 5s$ 7.5762	48 $^1S_0$ **Cd** Cadmium 112.414 [Kr]$4d^{10} 5s^2$ 8.9938	49 $^2P^\circ_{1/2}$ **In** Indium 114.818 [Kr]$4d^{10} 5s^2 5p$ 5.7864	50 $^3P_0$ **Sn** Tin 118.710 [Kr]$4d^{10} 5s^2 5p^2$ 7.3439	51 $^4S^\circ_{3/2}$ **Sb** Antimony 121.760 [Kr]$4d^{10} 5s^2 5p^3$ 8.6084	52 $^3P_2$ **Te** Tellurium 127.60 [Kr]$4d^{10} 5s^2 5p^4$ 9.0097	53 $^2P^\circ_{3/2}$ **I** Iodine 126.90447 [Kr]$4d^{10} 5s^2 5p^5$ 10.4513	54 $^1S_0$ **Xe** Xenon 131.293 [Kr]$4d^{10} 5s^2 5p^6$ 12.1298
6	55 $^2S_{1/2}$ **Cs** Cesium 132.9054520 [Xe]$6s$ 3.8939	56 $^1S_0$ **Ba** Barium 137.327 [Xe]$6s^2$ 5.2117		72 $^3F_2$ **Hf** Hafnium 178.49 [Xe]$4f^{14} 5d^2 6s^2$ 6.8251	73 $^4F_{3/2}$ **Ta** Tantalum 180.94788 [Xe]$4f^{14} 5d^3 6s^2$ 7.5496	74 $^5D_0$ **W** Tungsten 183.84 [Xe]$4f^{14} 5d^4 6s^2$ 7.8640	75 $^6S_{5/2}$ **Re** Rhenium 186.207 [Xe]$4f^{14} 5d^5 6s^2$ 7.8335	76 $^5D_4$ **Os** Osmium 190.23 [Xe]$4f^{14} 5d^6 6s^2$ 8.4382	77 $^4F_{9/2}$ **Ir** Iridium 192.217 [Xe]$4f^{14} 5d^7 6s^2$ 8.9670	78 $^3D_3$ **Pt** Platinum 195.084 [Xe]$4f^{14} 5d^9 6s$ 8.9588	79 $^2S_{1/2}$ **Au** Gold 196.966569 [Xe]$4f^{14} 5d^{10} 6s$ 9.2256	80 $^1S_0$ **Hg** Mercury 200.592 [Xe]$4f^{14} 5d^{10} 6s^2$ 10.4375	81 $^2P^\circ_{1/2}$ **Tl** Thallium 204.38* [Hg]$6p$ 6.1083	82 $^3P_0$ **Pb** Lead 207.2 [Hg]$6p^2$ 7.4167	83 $^4S^\circ_{3/2}$ **Bi** Bismuth 208.98040 [Hg]$6p^3$ 7.2855	84 $^3P_2$ **Po** Polonium (209) [Hg]$6p^4$ 8.414	85 $^2P^\circ_{3/2}$ **At** Astatine (210) [Hg]$6p^5$ 9.31751	86 $^1S_0$ **Rn** Radon (222) [Hg]$6p^6$ 10.7485
7	87 $^2S_{1/2}$ **Fr** Francium (223) [Rn]$7s$ 4.0727	88 $^1S_0$ **Ra** Radium (226) [Rn]$7s^2$ 5.2784		104 $^3F_2$ **Rf** Rutherfordium (267) [Rn]$5f^{14} 6d^2 7s^2$ 6.01	105 $^4F_{3/2}$ **Db** Dubnium (268) [Rn]$5f^{14} 6d^3 7s^2$ 6.8	106 **Sg** Seaborgium (271) [Rn]$5f^{14} 6d^4 7s^2$ 7.8	107 **Bh** Bohrium (272) [Rn]$5f^{14} 6d^5 7s^2$ 7.7	108 **Hs** Hassium (270) [Rn]$5f^{14} 6d^6 7s^2$ 7.6	109 **Mt** Meitnerium (276)	110 **Ds** Darmstadtium (281)	111 **Rg** Roentgenium (280)	112 **Cn** Copernicium (285)	113 **Uut** Ununtrium (284)	114 **Fl** Flerovium (289)	115 **Uup** Ununpentium (288)	116 **Lv** Livermorium (293)	117 **Uus** Ununseptium (294)	118 **Uuo** Ununoctium (294)

Lanthanides	57 $^2D_{3/2}$ **La** Lanthanum 138.90547 [Xe]$5d6s^2$ 5.5769	58 $^1G^\circ_4$ **Ce** Cerium 140.116 [Xe]$4f5d6s^2$ 5.5386	59 $^4I^\circ_{9/2}$ **Pr** Praseodymium 140.907 [Xe]$4f^3 6s^2$ 5.473	60 $^5I_4$ **Nd** Neodymium 144.242 [Xe]$4f^4 6s^2$ 5.5250	61 $^6H^\circ_{5/2}$ **Pm** Promethium (145) [Xe]$4f^5 6s^2$ 5.582	62 $^7F_0$ **Sm** Samarium 150.36 [Xe]$4f^6 6s^2$ 5.6437	63 $^8S^\circ_{7/2}$ **Eu** Europium 151.964 [Xe]$4f^7 6s^2$ 5.6704	64 $^9D^\circ_2$ **Gd** Gadolinium 157.25 [Xe]$4f^7 5d6s^2$ 6.1498	65 $^6H^\circ_{15/2}$ **Tb** Terbium 158.92535 [Xe]$4f^9 6s^2$ 5.8638	66 $^5I_8$ **Dy** Dysprosium 162.500 [Xe]$4f^{10} 6s^2$ 5.9391	67 $^4I^\circ_{15/2}$ **Ho** Holmium 164.93033 [Xe]$4f^{11} 6s^2$ 6.0215	68 $^3H_6$ **Er** Erbium 167.259 [Xe]$4f^{12} 6s^2$ 6.1077	69 $^2F^\circ_{7/2}$ **Tm** Thulium 168.93422 [Xe]$4f^{13} 6s^2$ 6.1843	70 $^1S_0$ **Yb** Ytterbium 173.054 [Xe]$4f^{14} 6s^2$ 6.2542	71 $^2D_{3/2}$ **Lu** Lutetium 174.9668 [Xe]$4f^{14} 5d6s^2$ 5.4259
Actinides	89 $^2D_{3/2}$ **Ac** Actinium (227) [Rn]$6d7s^2$ 5.3802	90 $^3F_2$ **Th** Thorium 232.0377 [Rn]$6d^2 7s^2$ 6.3067	91 $^4K_{11/2}$ **Pa** Protactinium 231.03588 [Rn]$5f^2 6d7s^2$ 5.89	92 $^5L^\circ_6$ **U** Uranium 238.02891 [Rn]$5f^3 6d7s^2$ 6.1941	93 $^6L_{11/2}$ **Np** Neptunium (237) [Rn]$5f^4 6d7s^2$ 6.2655	94 $^7F_0$ **Pu** Plutonium (244) [Rn]$5f^6 7s^2$ 6.0258	95 $^8S^\circ_{7/2}$ **Am** Americium (243) [Rn]$5f^7 7s^2$ 5.9738	96 $^9D^\circ_2$ **Cm** Curium (247) [Rn]$5f^7 6d7s^2$ 5.9914	97 $^6H^\circ_{15/2}$ **Bk** Berkelium (247) [Rn]$5f^9 7s^2$ 6.1978	98 $^5I_8$ **Cf** Californium (251) [Rn]$5f^{10} 7s^2$ 6.2817	99 $^4I^\circ_{15/2}$ **Es** Einsteinium (252) [Rn]$5f^{11} 7s^2$ 6.3676	100 $^3H_6$ **Fm** Fermium (257) [Rn]$5f^{12} 7s^2$ 6.50	101 $^2F^\circ_{7/2}$ **Md** Mendelevium (258) [Rn]$5f^{13} 7s^2$ 6.58	102 $^1S_0$ **No** Nobelium (259) [Rn]$5f^{14} 7s^2$ 6.65	103 $^2P^\circ_{1/2}$ **Lr** Lawrencium (262) [Rn]$5f^{14} 7s^2 7p$ 4.90

Key:

Atomic Number	58
Ground-state Level	$^1G^\circ_4$
Symbol	**Ce**
Name	Cerium
Standard Atomic Weight†	140.116
Ground-state Configuration	[Xe]$4f5d6s^2$
Ionization Energy (eV)	5.5386

†Based upon ^{12}C. () indicates the mass number of the longest-lived isotope.

*IUPAC conventional atomic weights; standard atomic weights for these elements are expressed in intervals; see iupac.org for an explanation and values.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2014)

CNO Bi-Cycle

In higher mass stars, most hydrogen is fused via the CNO by-cycle. This chain fuses hydrogen to helium using CNO as a catalyst.

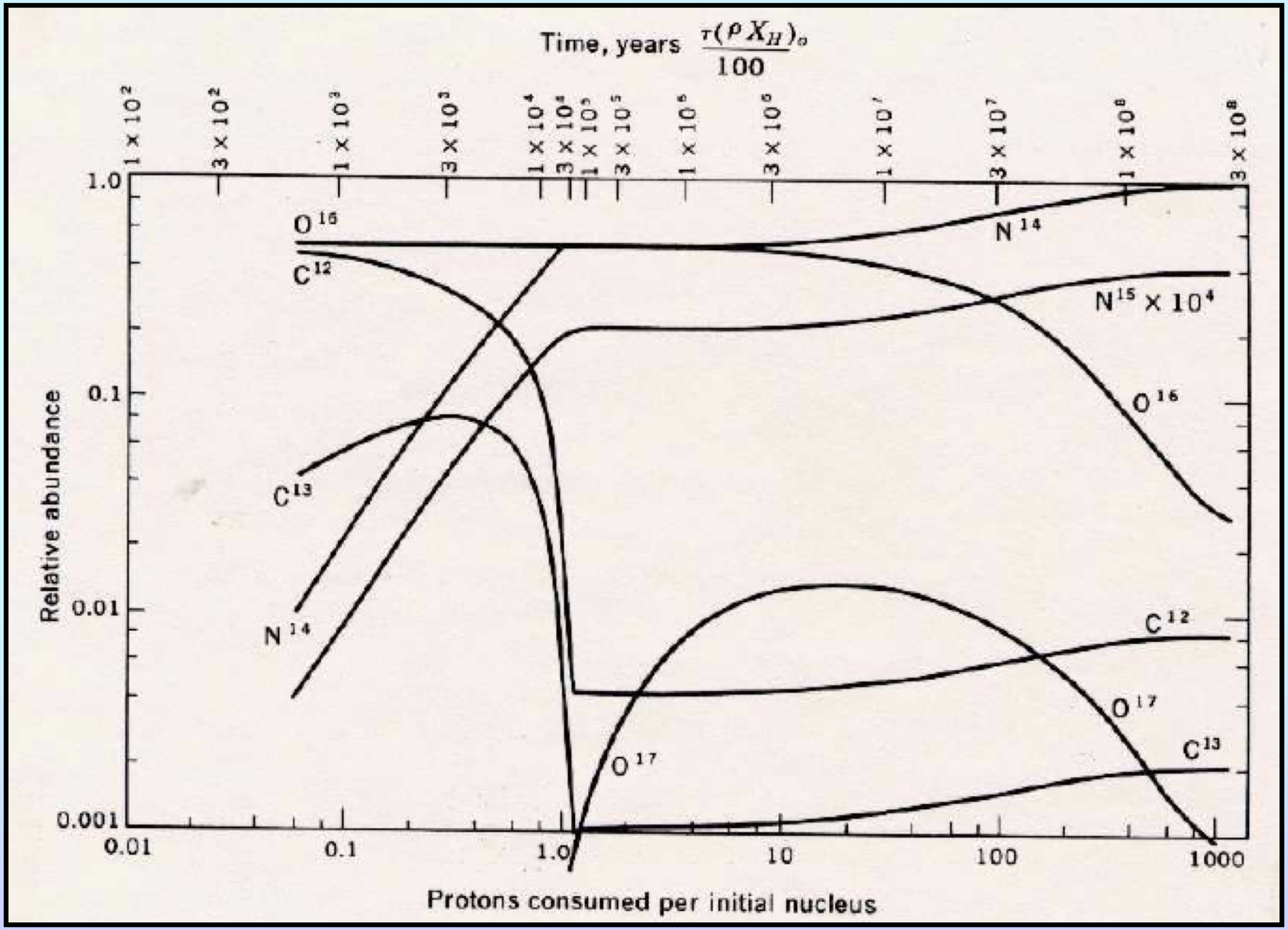
- $^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$
- $^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$ $\langle\varepsilon_\nu\rangle = 0.71$ MeV
- $^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$
- $^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$
- $^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$ $\langle\varepsilon_\nu\rangle = 1.00$ MeV
- $^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$

or (once every ~ 2500 times)

- $^{15}N + {}^{1}H \rightarrow {}^{16}O + \gamma$
- $^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$
- $^{17}F \rightarrow {}^{17}O + e^{+} + \nu_e$ $\langle\varepsilon_\nu\rangle = 0.94$ MeV
- $^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$

CNO Bi-Cycle

Note: the net effect of the CNO cycle is 4 $^1H \rightarrow {}^4He$. In the process, the relative abundances of CNO get changed. In particular, since ^{14}N has the smallest cross section, it tends to get built up, while C is lost.

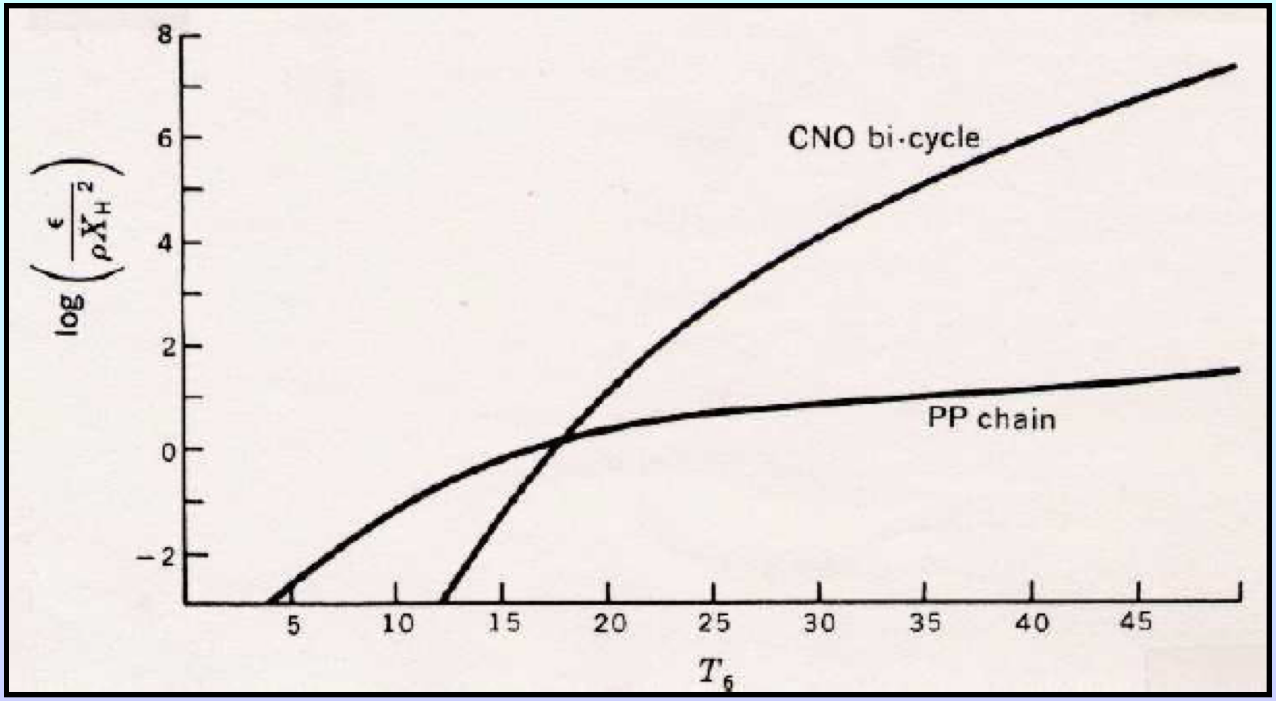


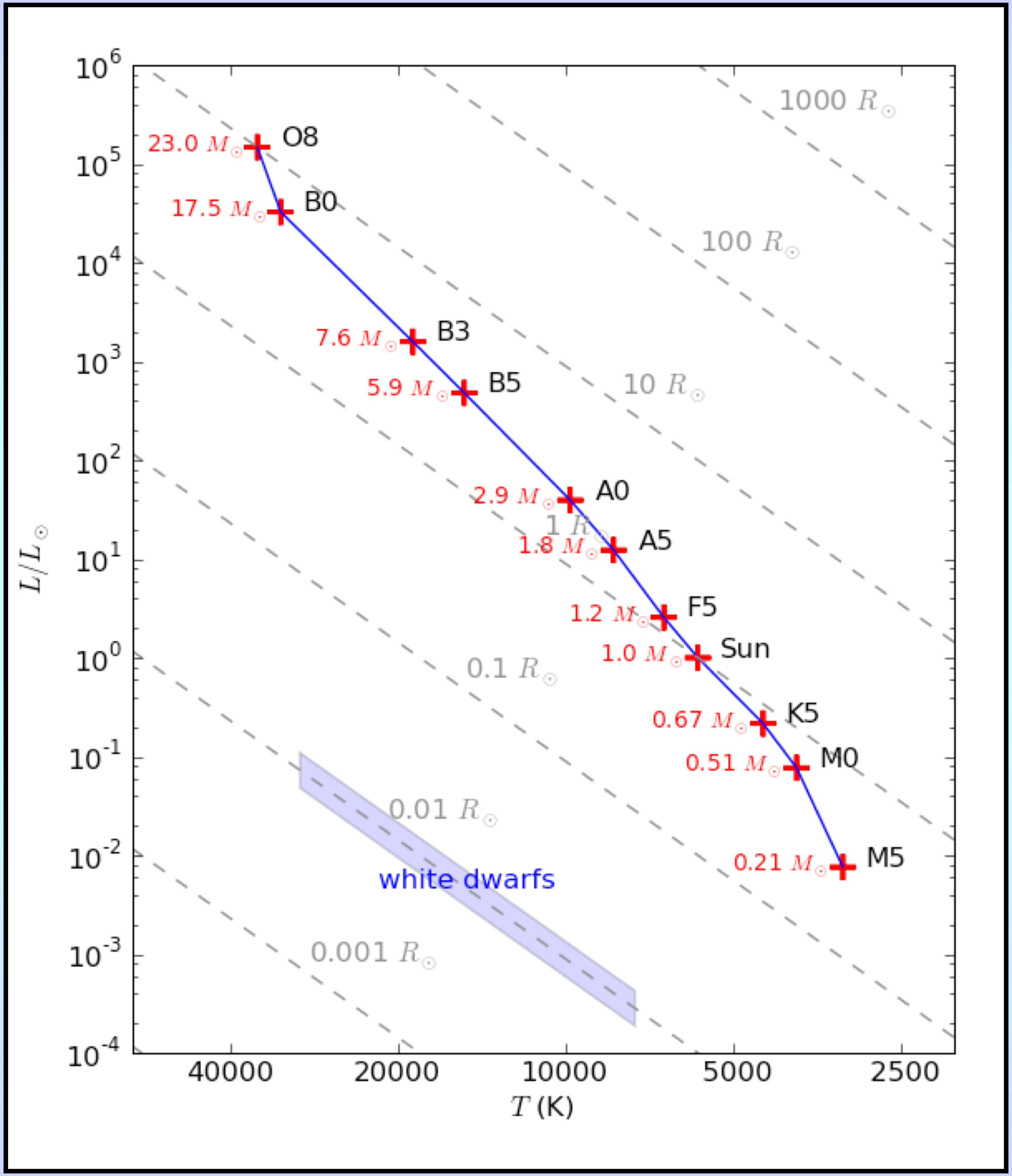
Nuclear Reaction Rates

Since the proton-proton chain involves ions with the least amount of charge, it has the smallest temperature dependence of any reaction.

$$\varepsilon_{pp} \propto \rho X^2 e^{-33.80/T_6^{1/3}} \text{ ergs s}^{-1}\text{ cm}^{-3}$$

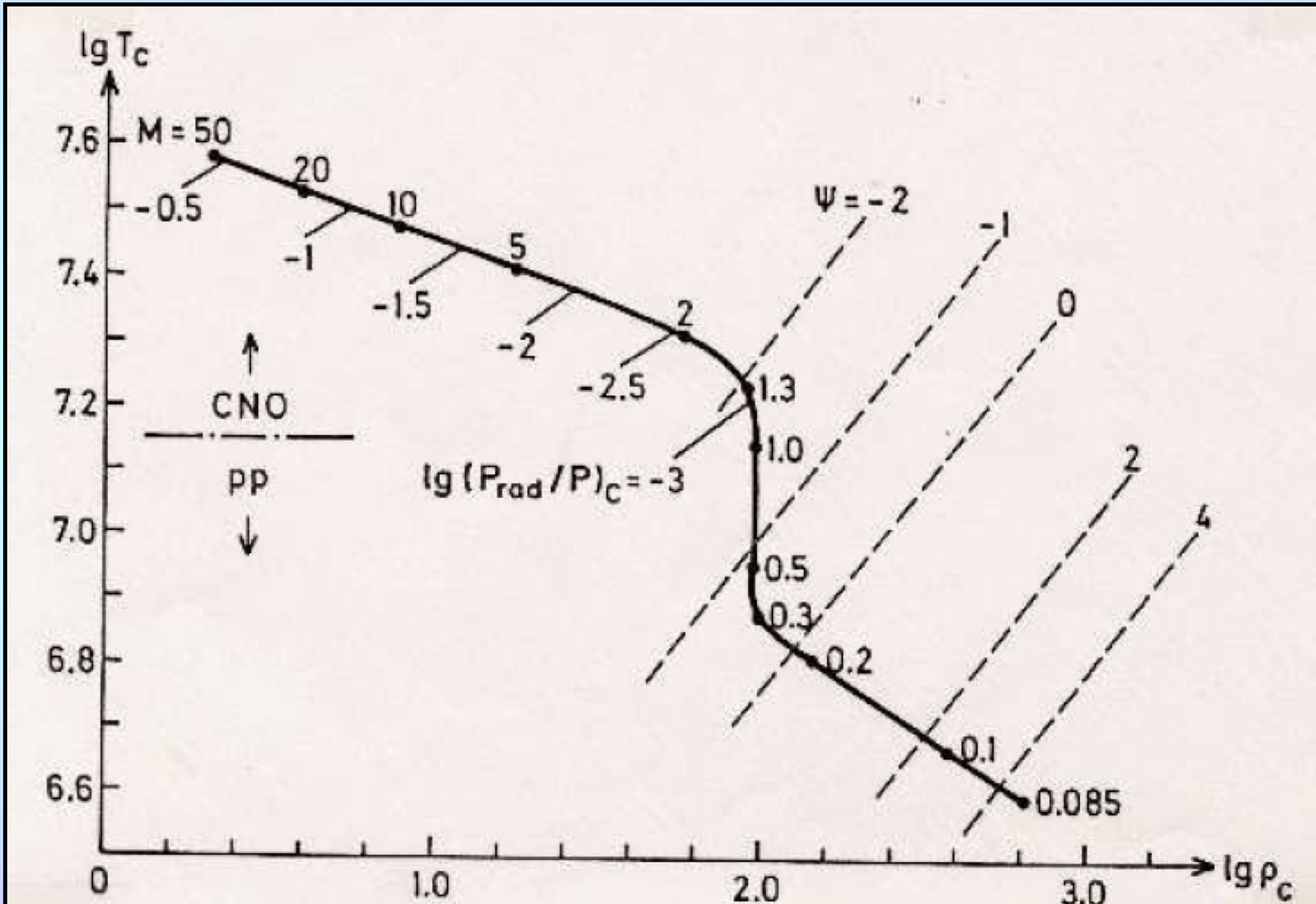
Thus, at $T_6 \sim 5$, $\varepsilon_{pp} \propto T^6$, while at $T_6 \sim 20$, $\varepsilon_{pp} \propto T^{3.5}$. Because CNO ions have more electrostatic repulsion, the temperature dependence of its process is stronger, $\varepsilon_{CNO} \propto T^{23}$ at $T_6 \sim 10$ to $\varepsilon_{CNO} \propto T^{13}$ at $T_6 \sim 50$.



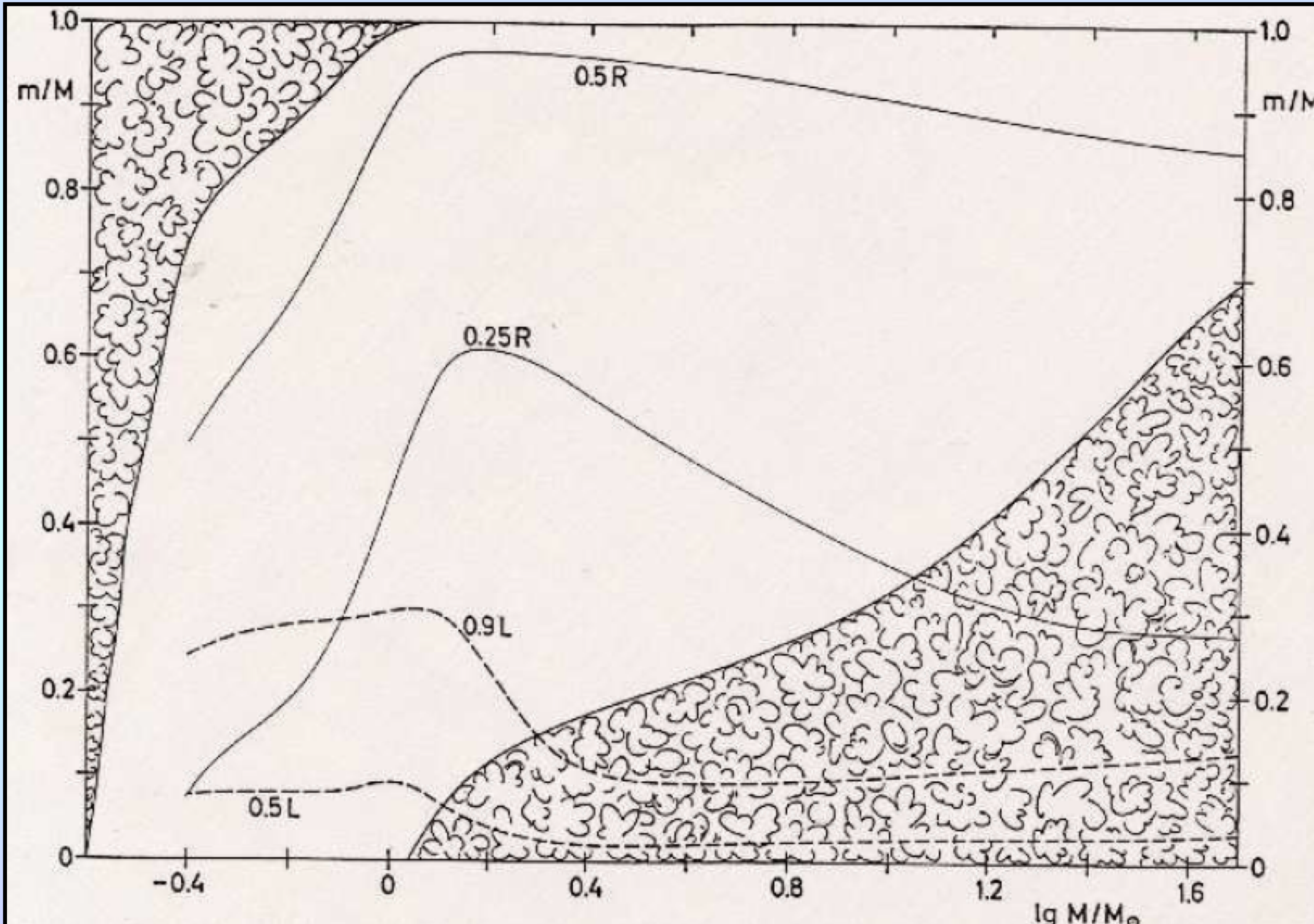


The HR Diagram

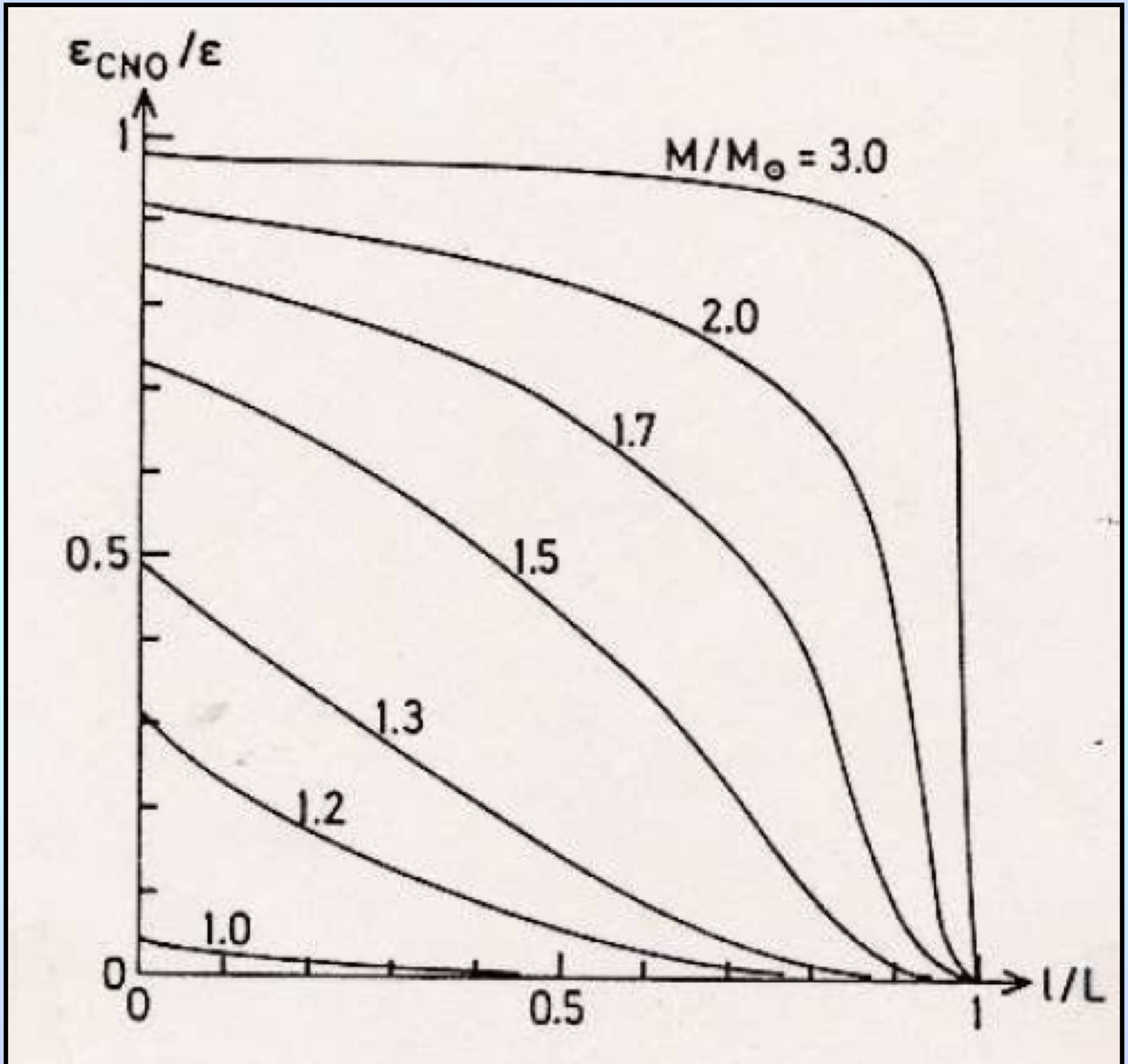
The Central Temperature and Density for Main Sequence Stars



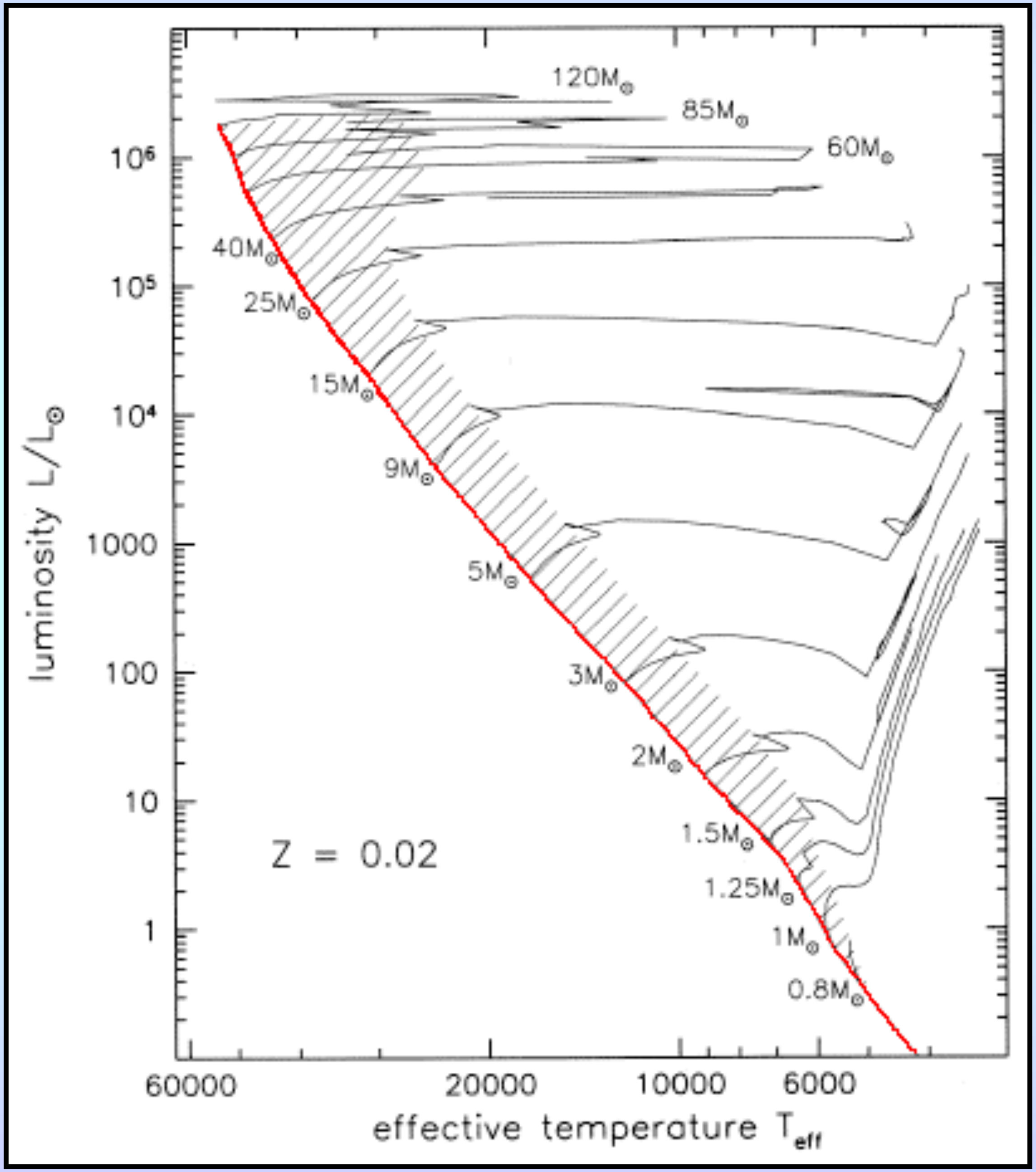
Convection on the Main Sequence



Importance of CNO Burning versus Stellar Mass



Location of the Main Sequence



In general, the opacity of a star is proportional to its metal abundance (due to bound-free transitions and electrons supplied to H^-.) The lower the metal abundance, the smaller the opacity, the less energy is trapped in the star doing work, the smaller the star, and therefore the hotter the star. The metal-poor main sequence is bluer than the metal-rich main sequence.